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Effect of Harmonics' Modeling in Kalman Filter Performance

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Abstract: *In many of applications, a correct estimation of the fundamental component of measured signal is required, several filters are implemented for this purpose such as Kalman filter. To estimate the fundamental component using kalman filter, you need only two variables to represent it in kalman model, the effect of modeling the harmonics on the accuracy of the kalman filter of the fundamental component estimation, was investigated in this paper. Where the results show that including the harmonics in the kalman model enhance the fundamental component estimation. Since we interest in estimating the fundamental component, there will be no need to estimate the individual values of the harmonics, especially that estimating one order of harmonics increases the number of state variables by , and most of the time the order of harmonis in the input signal can't be recognized, so in this case it is more important to estimate the total harmonics in the signal, based on that a noval model of total harmonic components of the input signal is proposed, where the first 15 harmonics were modeled by using 6 state variables, and the results showed that the proposed model has better results than the conventional models.*

Keyword: *Fundamental component, Harmonics, Kalman Filter, Modeling.*

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I. Introduction

In power system, a lot of protection and control devices need to estimate the exact value of the fundamental signal, to work correctly, where the current and voltage suffer from the harmonics due to power electronics circuit, switches, nonlinear load and Photovoltaic cell ..etc [1]. Kalman filter is commonly used to estimate the fundamental component in power system [2], where modeling the fundamental signal is so simple and required only two state variables. several previous researches combined the kalman filter with other optimization methods to enhance kalman filter performance and other researches proposed adaptive techniques based on kalman filter to overcome uncertainty of some parameters in kalman model [3-16]. Kalman first estimates the output using the system model, then the estimated value is updated based on the error between the estimated value and the measurement value[17], so if the model includes only the fundamental component, the error in this case will be large, since it contains the difference between the actual fundamental signal and the estimated signal, beside all the harmonics in the measured signal. Since the estimation output is updated based on the error signal, it will be better, if the error signal contains only the difference between the estimated value and the measured value, this can be done by modeling the signal harmonics in kalman filter model.It is so difficult to determine which harmonics must be modeled prior measurement process, and the order of harmonics may be vary through the measurement process, due to nonlinear loads and switches, so the effect of modeling the harmonics in kalman filter will be discussed in this paper.To include one order of harmonic in kalman model needs two state variables, and this will increase the size of the model matrices, and increase the number of computation , and since the purpose of kalman filter in most of applications is the fundamental signal estimation, so modeling the harmonics will be included only to enhance this estimation, which means the individual values of harmonics component are not needed, what is needed is the total harmonics in the signal (summation of all the harmonics component), for that reason a novel model of the summation of the first 15 harmonics by using only 6 state variables is proposed

II. Kalman Filter

Kalman filter is a recursive linear optimal filter, to estimate the state variables and the output of a system modeled as follow[17]:

$$X_{k/k} = A_k X_{k/k-1} + B_k U_k + W_k$$
$$Y_k = C_k X_{k/k} + D_k U_k + V_k$$

Where:

A_k : transition matrix.

B_k : input control vector.

W_k : process noise.

Y_k : observation state vector.

C_k : observation matrix.

V_k : observation noise.

the estimation can be divided into two stages; predicted stage based on the system model and updated stage based on the measurement signal, the mathematical equation of these states are follows:

- Predicted stage:

$$X_{k/k-1} = A_k X_{k-1/k-1} + B_{k-1} U_{k-1}$$

$$P_{k/k-1} = A_k P_{k-1/k-1} + A_k^T + Q_k$$

- Updating Stage:

$$Y_k = Z_k - C_k X_{k/k-1}$$

$$K_k = P_{k/k-1} C_k^T [C_k P_{k/k-1} C_k^T + R_k]^{-1}$$

$$X_{k/k} = X_{k/k-1} + K_k Y_k$$

$$P_{k/k} = [1 - K_k C_k] P_{k/k-1}$$

Where:

Q_k : process noise covariance matrix.

R_k : observation noise covariance matrix.

Effect of including harmonics in kalman filter model

Let us first illustrate the behavior of kalman filter for fundamental frequency detection using simple example, by assuming the input signal as follows:

$$y_{input} = 10\cos(100\pi t) + 8\cos(150\pi t) + 6\cos(200\pi t) + 4\cos(300\pi t)$$

The input signal y_{input} has fundamental, third, fourth and sixth harmonic components, to show the effect of including the harmonics in the Kalman filter model, four models were implemented here, the first one (Model 1)contain the fundamental component only, the second model (Model2) contains the fundamental and the third harmonic, the third model (Model3) contains the fundamental, third and the fourth harmonics, the last model (Model4) contains the fundamental and all the harmonics in the input signal, the result of the error signal (i.e error between the fundamental signal in the input signal and the estimated fundamental signal by kalman filter) and the estimated amplitude of the four models are shown in Fig.1 and Fig.2 respectively, it is clear that the best results are obtained when the kalman model contains all the harmonics that are existed in the input signal.

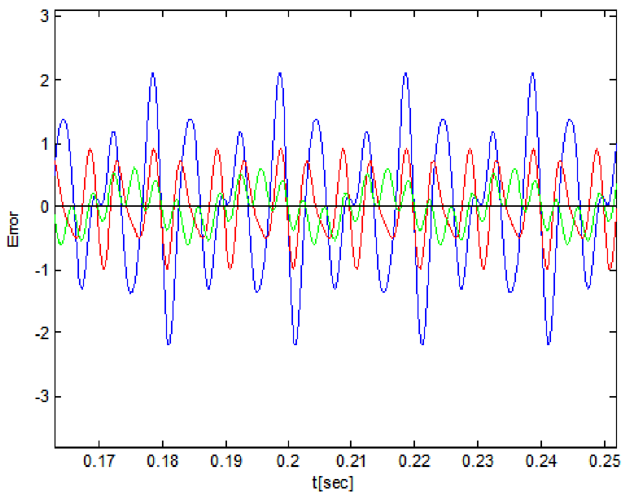


Fig.1 Error signal of the four models; Model1 (blue), Model2 (red), Model3 (green) and Model4 (black)

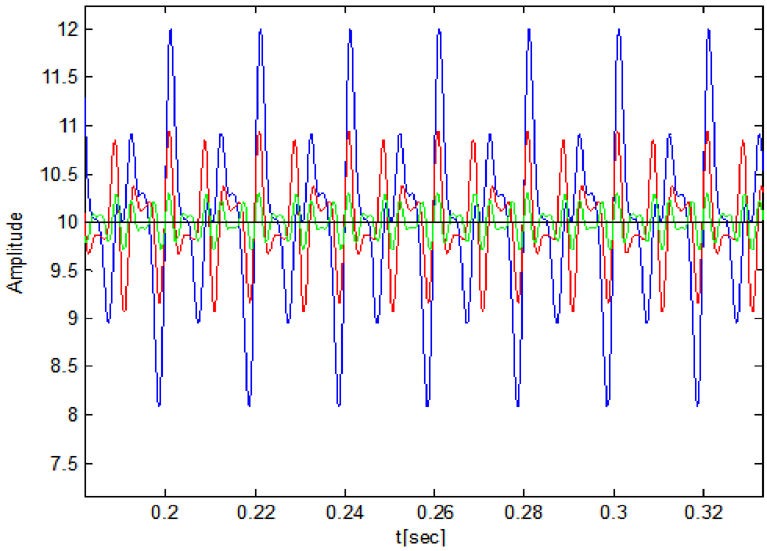


Fig.2 Amplitude of the fundamental signal or the four models; Model1 (blue), Model2 (red), Model3 (green) and Model4 (black)

Most of the times we don't know the order of the harmonics in the input signal, so now another models will be implemented to show if including extra harmonics in the kalman model, (where these harmonics are not existed in the input signal), affect the performance of the kalman filter. Model1 now will be compared with Model5, Model6 and Model7 where all Models contains the fundamental component and Model5 contain 2nd harmonic, Model6 contains 2nd and 5th harmonic, Model7 contains 2nd, 5th and 7th harmonics, the main feature here in these models, that the order of harmonics implemented in these models are not existed in the input signal, and neither of the harmonic components in the input signal are included in these models. The results of the error signal and the estimated amplitude of the fundamental signal are shown in Fig.3 and Fig.4 respectively, the output of these models are almost the same, where including extra harmonics that are not existed in the input signal in the kalman model doesn't affect the performance of the kalman filter. To make sure about this result, Model2 is also compared with other models that have extra harmonics beside the fundamental and third order of harmonics, where these extra harmonics are not existed in the input signal, where Model8 contains 2nd, Model9 contains 2nd and 5th harmonics, Model10 contains 2nd, 5th and 7th harmonics, the results of the error signal and the amplitude of the estimated fundamental signal are shown in Fig.5 and Fig.6 respectively, again from these figures it is clear that the theses models don't have too much difference. The last comparison will be done here to show that including extra harmonics in the kalman filter (that are not existed in the input signal) doesn't affect the kalman filter performance is to compare Model4 which contains all the harmonics components of the input signal with Model 11 which has the same harmonics components of Model 4 and also has the 7th harmonic, the error signal and the amplitude of the estimated fundamental signal of Model 4 and Model 11 are shown in Fig.7 and Fig.8 respectively, in these figures you can't distinguish between the red and the blue lines, where the same result can be concluded here, that including extra harmonics which are not existed in the input signal doesn't affect the kalman filter performance.

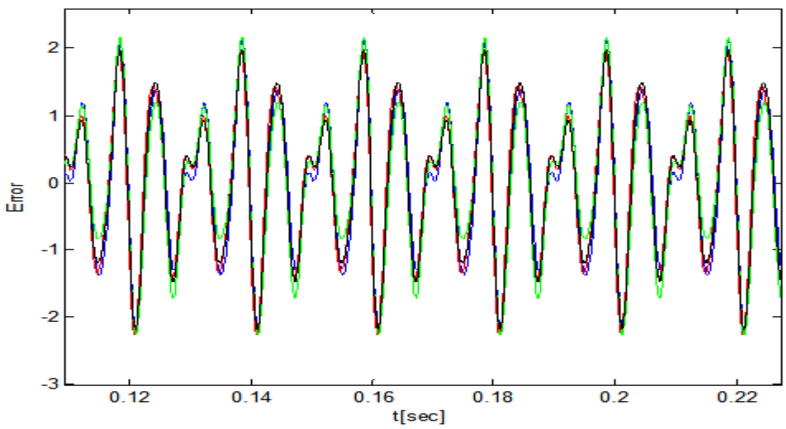


Fig.3 Error signal of Model1 (blue), Model5 (red), Model6 (green) and Model7 (black)

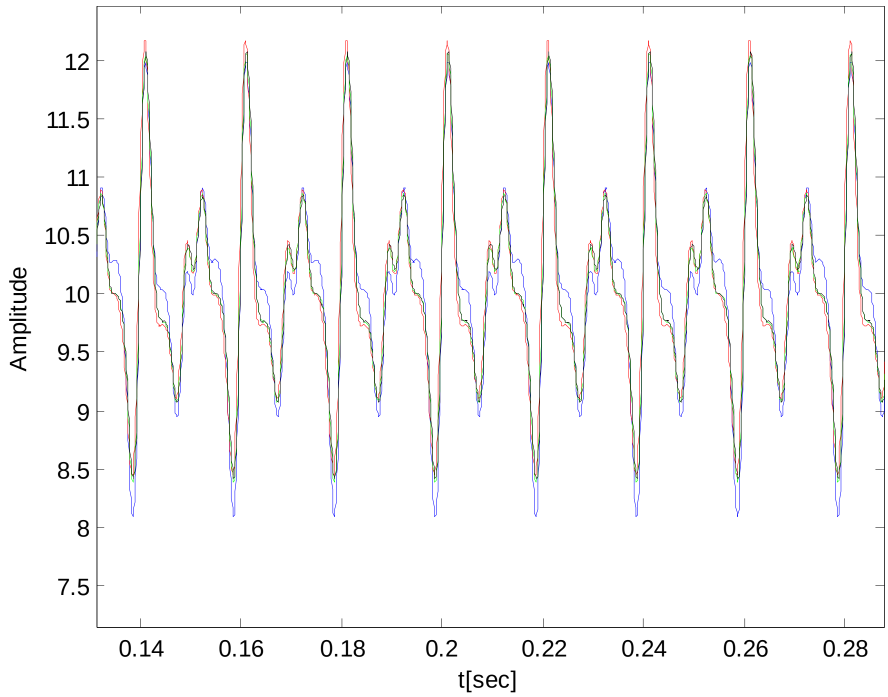


Fig.4 Amplitude of the fundamental signal of Model1 (blue), Model5 (red), Model6 (green) and Model7 (black)

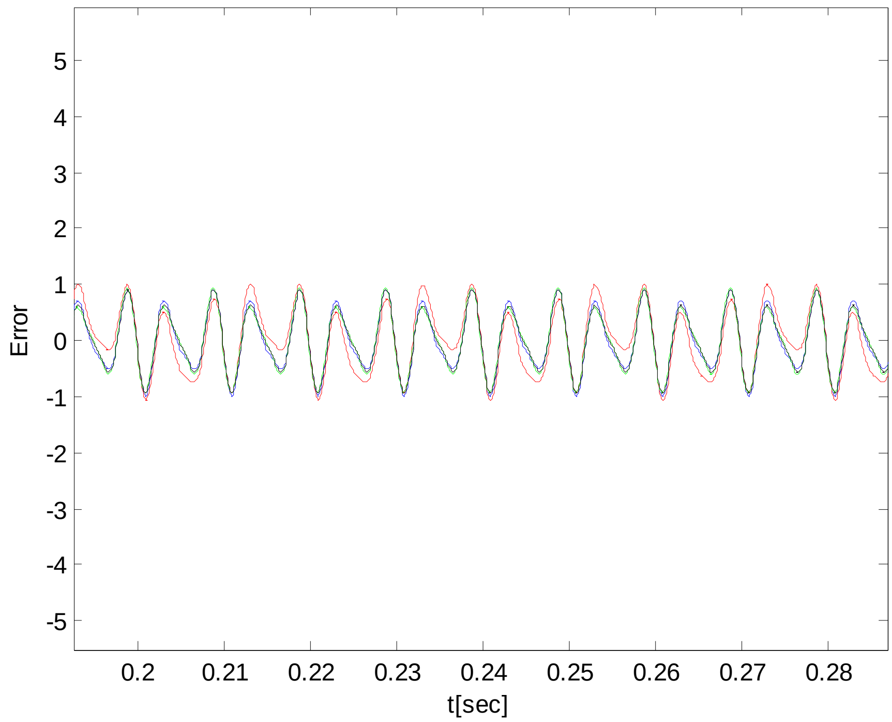


Fig.3 Error signal of Model2 (blue), Model8 (red), Model9 (green) and Model10 (black)

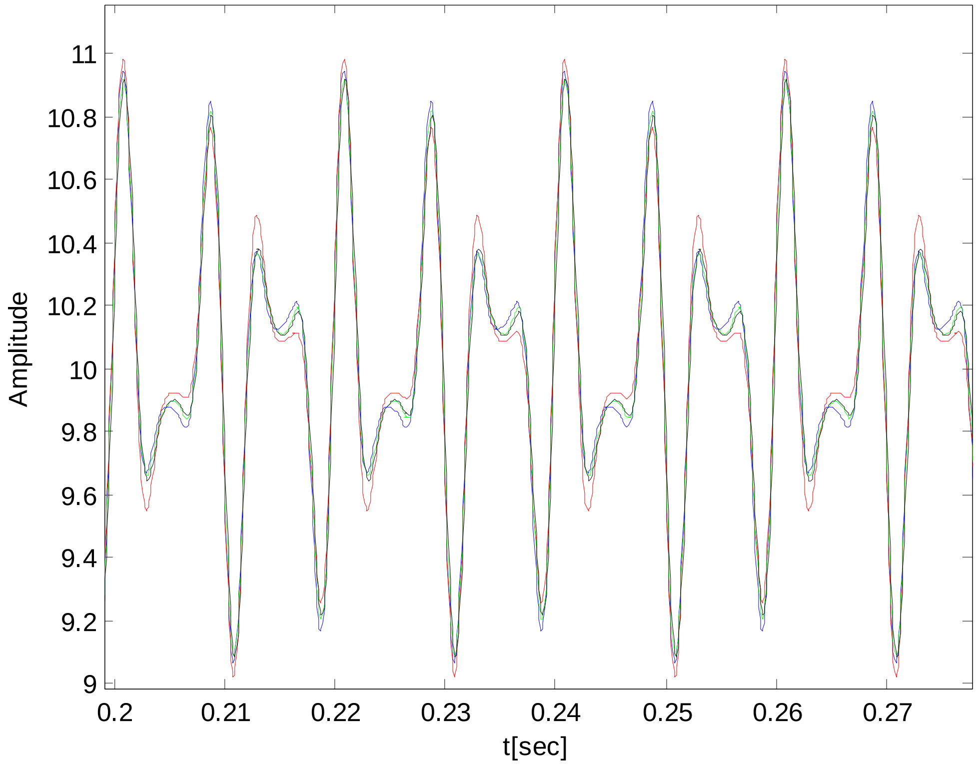


Fig.4 Amplitude of the fundamental signal of Model2 (blue), Model8 (red), Model9 (green) and Model10 (black)

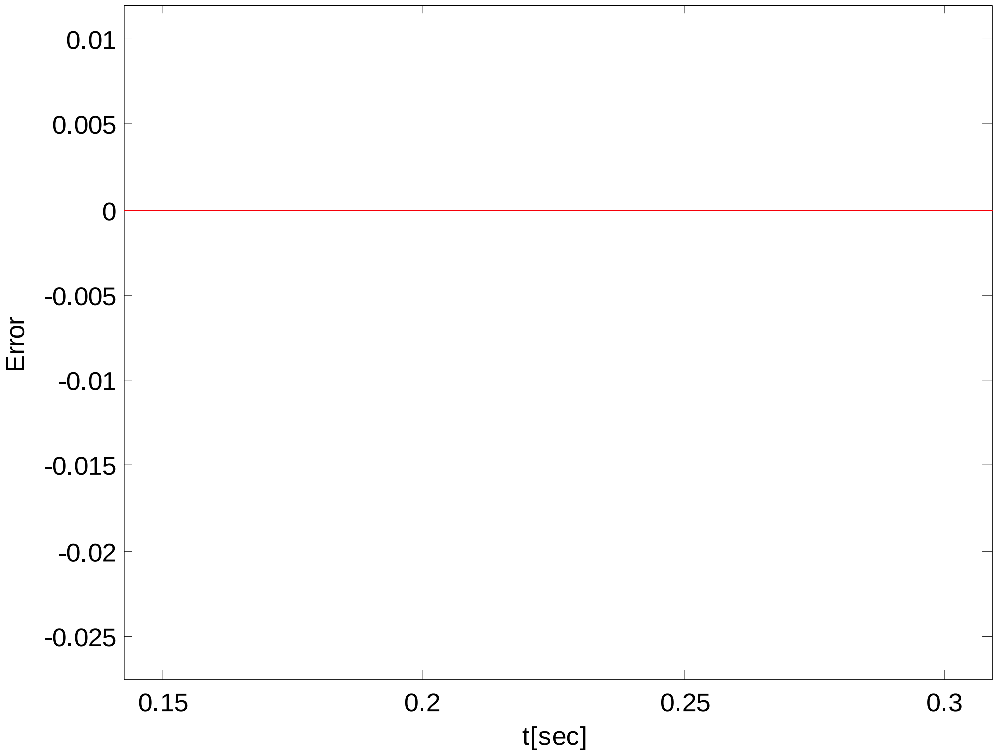


Fig.5 Error signal of Model4 (blue) and Model11 (red)

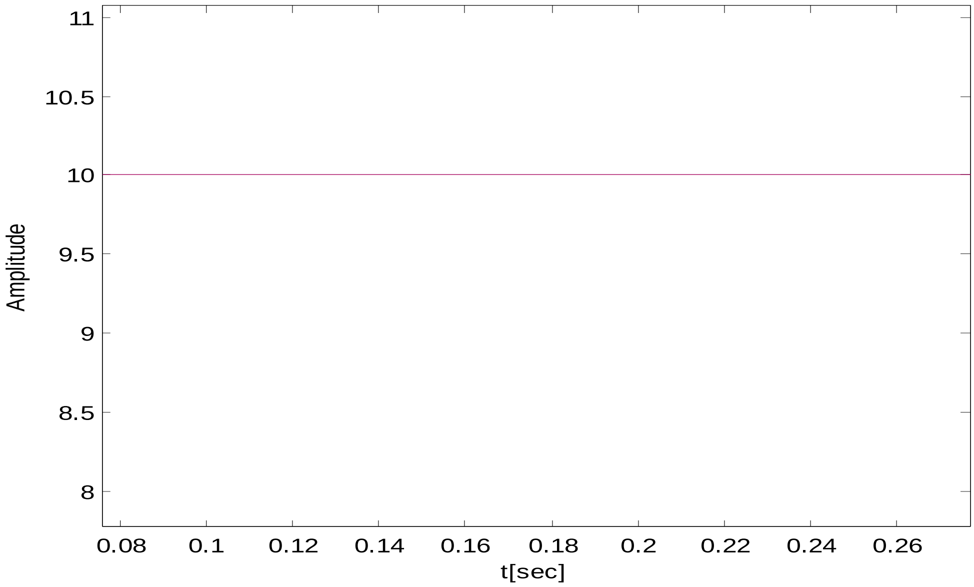


Fig.6 Amplitude of the fundamental signal of Model4 (blue) and Model11 (red).

III. Modeling the harmonics

It has been shown in the previous section that, including the harmonics in kalman filter model improve the performance of the kalman filter. if they are existed in the input signal and doesn't affect if they are not existed in the input signal, based on this result if the kalman filter model contains all the orders of the harmonics, this will lead to the best performance of the kalman filter, but including all the harmonics will increase the size of the kalman matrices and the number of calculations. Since the purpose of the kalman filter in this paper is to estimate the fundamental component, and modeling the harmonics is only used to improve the kalman output, so it isn't necessary to model the individual values of the harmonic component, so in the propsed model the signal can be divided in to two parts; the fundamental component and the harmonic component as follows:

$$y_{input} = y_f + y_h$$

Where $y_f = A_1 \cos(wt + \theta_1)$ and y_h must include all the harmonic components in the signal and it can be defined as a summation of all the harmonic components as follows:

$$y_h = \sum_{n=2}^{m} \cos(nwt + \theta_n)$$

So, now instead of modeling each harmonic components, it needs to model the summation of the harmonic components (y_h), 6 state variables were proposed to represent the summation of the first 15 harmonics, where if these 15 harmonics were modeled using the traditional way it required 30 state variables (the reason of choosing the first 15 harmonics will be illustrated later).

The proposed state variables for the harmonics will be as follow:

$$x_1(k) = \sum_{n=2}^{15} A_n \cos(nwkT + \theta_n)$$

$$x_2(k) = \sum_{n=2}^{15} A_n \cos(nwT)\cos(nwkT + \theta_n)$$

$$x_3(k) = \sum_{n=2}^{15} A_n \sin(nwT)\cos(nwkT + \theta_n)$$

$$x_4(k) = \sum_{n=2}^{15} A_n \sin(nwkT + \theta_n)$$

$$x_5(k) = \sum_{n=2}^{15} A_n \cos(nwT)\sin(nwkT + \theta_n)$$

$$x_6(k) = \sum_{n=2}^{15} A_n \sin(nwT)\sin(nwkT + \theta_n)$$

Now, the next sample X (k+1) of the state variables should be evaluated in terms of the current state variable X(k). It is easy to estimate the next sample of the first and the 4th variable in terms of the current state variables as follows :

$$x_1(k+1) = \sum_{n=2}^{15} A_n \cos(nw(k+1)T + \theta_n) = \sum_{n=2}^{15} A_n \cos(nwkT + \theta_n)\cos(nwT) - A_n \sin(nwkT + \theta_n)\sin(nwT)$$

$$x_1(k+1) = \sum_{n=2}^{15} A_n \cos(nwT)\cos(nwkT + \theta_n) - \sum_{n=2}^{15} A_n \sin(nwT)\sin(nwkT + \theta_n)$$

$$x_1(k+1) = x_3(k) - x_6(k)$$

$$x_4(k+1) = \sum_{n=2}^{15} A_n \sin(nw(k+1)T + \theta_n) = \sum_{n=2}^{15} A_n \sin(nwkT + \theta_n)\cos(nwT) + A_n \cos(nwkT + \theta_n)\sin(nwT)$$

$$x_4(k+1) = \sum_{n=2}^{15} A_n \cos(nwT)\sin(nwkT + \theta_n) + \sum_{n=2}^{15} A_n \sin(nwT)\cos(nwkT + \theta_n)$$

$$x_4(k+1) = x_5(k) + x_3(k)$$

For the other state variable:

$$x_2(k+1) = \sum_{n=2}^{15} A_n \cos(nwT)\cos(nw(k+1)T + \theta_n)$$

$$= \sum_{n=2}^{15} A_n \cos^2(nwT)\cos(nwkT + \theta_n) - \sum_{n=2}^{15} A_n \cos(nwT)\sin(nwT)\sin(nwkT + \theta_n)$$

$$x_3(k+1) = \sum_{n=2}^{15} A_n \sin(nwT)\cos(nw(k+1)T + \theta_n)$$

$$= \sum_{n=2}^{15} A_n \cos(nwT)\sin(nwT)\cos(nwkT + \theta_n) - \sum_{n=2}^{15} A_n \sin^2(nwT)\sin(nwkT + \theta_n)$$

$$x_5(k+1) = \sum_{n=2}^{15} A_n \cos(nwT)\sin(nw(k+1)T + \theta_n)$$

$$= \sum_{n=2}^{15} A_n \cos^2(nwT)\sin(nwkT + \theta_n) + \sum_{n=2}^{15} A_n \cos(nwT)\sin(nwT)\cos(nwkT + \theta_n)$$

$$x_6(k+1) = \sum_{n=2}^{15} A_n \sin(nwT)\sin(nw(k+1)T + \theta_n)$$

$$= \sum_{n=2}^{15} A_n \cos(nwT)\sin(nwT)\sin(nwkT + \theta_n) + \sum_{n=2}^{15} A_n \sin^2(nwT)\cos(nwkT + \theta_n)$$

Now let us plot the figures of $\cos^2(nwT), \sin(nwT), \cos(nwT)\sin(nwT)$ and $\sin^2(nwT)$ in terms of $\cos(nwT)$ or $\sin(nwT)$, the plot for n varies from 2 to 15, the results are shown in Fig.7, Fig.8 and Fig.9. using basic fitting these figures are linear and can be represented by a linear equation in terms of the x-axis signal as folllows:

$$\cos^2(nwT) = B_1 \cos(nwT) + B_2$$
$$\sin^2(nwT) = B_3 \cos(nwT) + B_4 \qquad \mathrm{n} = 1 \,\mathrm{to}\, 15$$
$$\cos(nwT)\sin(nwT) = B_5 \sin(nwT) + B_6$$

Where
$B_1 = 1.8961,\ \ B_2 = -0.89758\ ,\ B_3 = -1.8961,\ \ B_4 = 1.8976\ ,\ B_5 = 0.88033$ and $B_6 = 0.015997$

the main reason of modeling only the first 15 harmonics is to have a linear relation and to use linear curve fitting, exceed the number of harmonics to be included in the model will increase the error between the actual value and the linear equation, actually a linear relation can be extended up to 20 oredr of harmonics, so in future works, if it is needed to model more than 15 harmonics another state variables can be added to represent the next 15 harmonics.

Substitute these coefficient in the state variables equations will leads to:

$$\begin{aligned} x_2(k+1) &= B_1 \sum_{n=2}^{15} A_n \cos(nwT)\cos(nwkT+\theta_n) + B_2 \sum_{n=2}^{15} A_n \cos(nwkT+\theta_n) \\ &\quad - B_5 \sum_{n=2}^{15} A_n \sin(nwT)\sin(nwkT+\theta_n) - B_6 \sum_{n=2}^{15} A_n \sin(nwkT+\theta_n) \\ &= B_1 x_2(k) + B_2 x_1(k) - B_5 x_6(k) - B_6 x_4(k) \end{aligned}$$

$$\begin{aligned} x_3(k+1) &= B_5 \sum_{n=2}^{15} A_n \sin(nwT)\cos(nwkT+\theta_n) + B_6 \sum_{n=2}^{15} A_n \cos(nwkT+\theta_n) \\ &\quad - B_3 \sum_{n=2}^{15} A_n \cos(nwT)\sin(nwkT+\theta_n) - B_4 \sum_{n=2}^{15} A_n \sin(nwkT+\theta_n) \\ &= B_5 x_3(k) + B_6 x_1(k) - B_3 x_5(k) - B_4 x_4(k) \end{aligned}$$

$$\begin{aligned} x_5(k+1) &= B_1 \sum_{n=2}^{15} A_n \cos(nwT)\sin(nwkT+\theta_n) + B_2 \sum_{n=2}^{15} A_n \sin(nwkT+\theta_n) \\ &\quad + B_5 \sum_{n=2}^{15} A_n \sin(nwT)\cos(nwkT+\theta_n) + B_6 \sum_{n=2}^{15} A_n \cos(nwkT+\theta_n) \\ &= B_1 x_5(k) + B_2 x_4(k) + B_5 x_3(k) + B_6 x_1(k) \end{aligned}$$

$$\begin{aligned} x_6(k+1) &= B_5 \sum_{n=2}^{15} A_n \sin(nwT)\sin(nwkT+\theta_n) + B_6 \sum_{n=2}^{15} A_n \sin(nwkT+\theta_n) \\ &\quad + B_3 \sum_{n=2}^{15} A_n \cos(nwT)\cos(nwkT+\theta_n) + B_4 \sum_{n=2}^{15} A_n \cos(nwkT+\theta_n) \\ &= B_5 x_6(k) + B_6 x_4(k) + B_3 x_2(k) + B_4 x_1(k) \end{aligned}$$

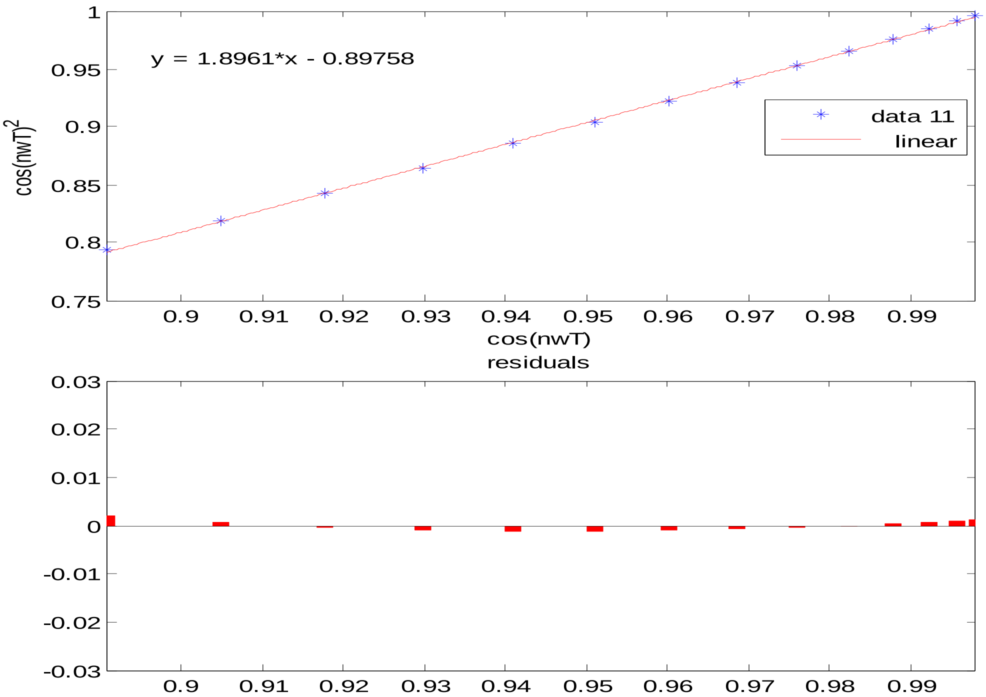


Fig.7 linear fitting of $\cos^2(wnT)$ in term of $\cos(nwT)$

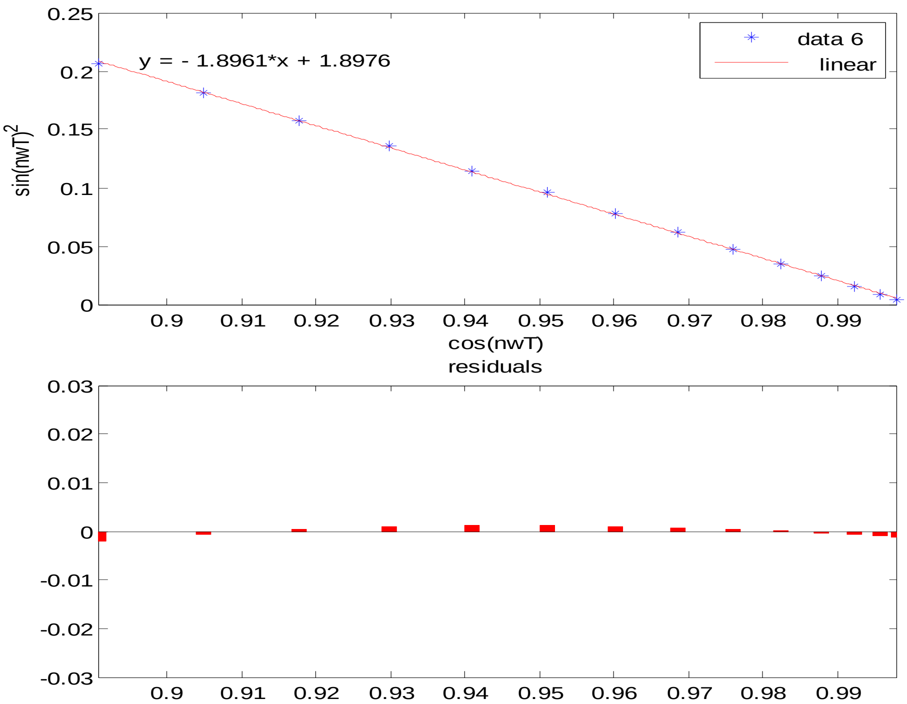


Fig.8 linear fitting of $\sin^2(wnT)$ in term of $\cos(nwT)$

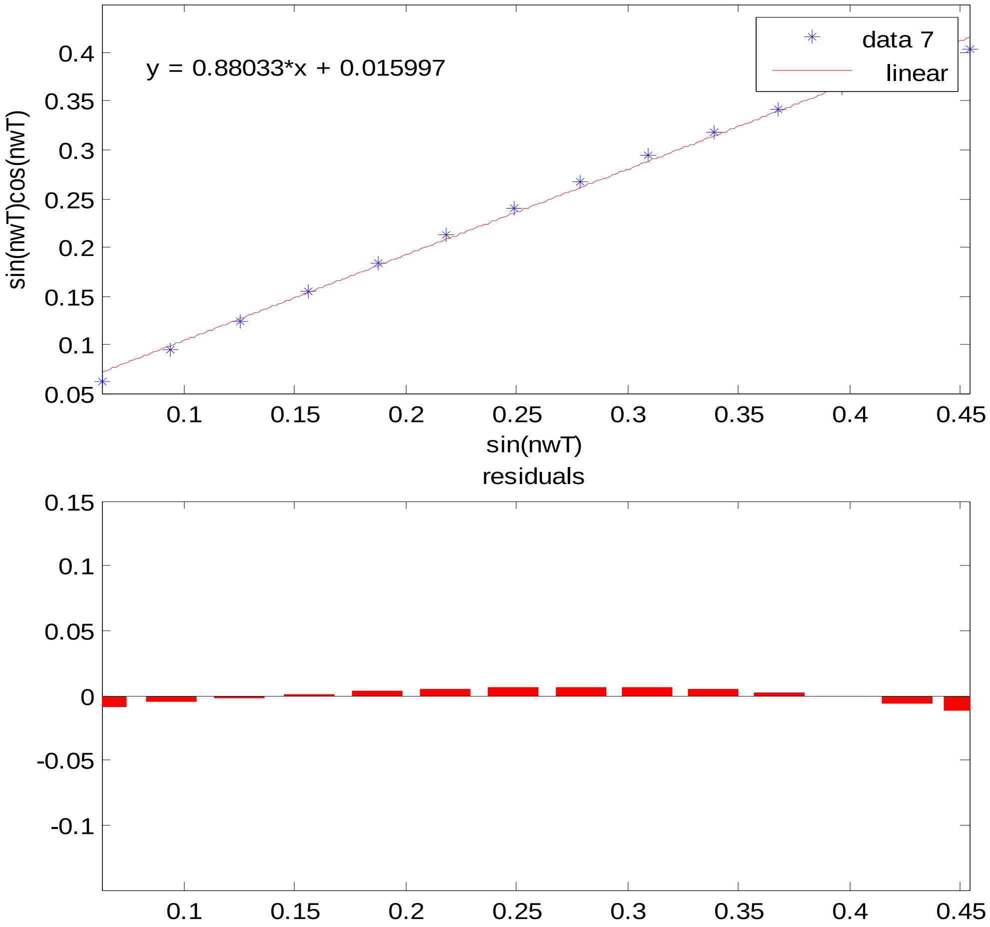


Fig.9 linear fitting of $\cos(nwT)\sin(wnT)$ in term of $\sin(nwT)$

IV. Results And Discussion

First, let us investigate the performance of the proposed model for an input signal contains 15 harmonics (the same order of harmonics of the proposed model), Fig.10, shows the fundamental component (red line) and the input signal (blue line) , and Fig.11 shows the harmonics components in the input signal using Fast Fourier Transform (FFT), the fundamental component will be estimated by the proposed model then compare the result with a Kaman model (Model12) which contains the first fourth harmonic, the number of state variables in the proposed model are 8 variables, where Model12 has 10 state variables,. The error signal of the proposed model (red line) and the error signal of Model12 (blue line) are shown in Fig.12, where it clear that the proposed model has better estimation. zThe Third state variable in the proposed model estimates the summation of all the harmonic components in the input signal, to verify this point, FFT was used to evaluate the harmonic components in this state variable (red line) and compare them with the harmonic component in the input signal (blue line), where the results are shown in Fig.13, where X_3 has the same harmonic components of the input signal, and the amplitude of some harmonics are the same where you can't distinguish between red bars from the blue bars.Now let us increase the order of harmonics of the input signal up to 40, for the same fundamental signal, the error signal of the proposed model and Model12 is shown in Fig.14. FFT of X_3 of the proposed model and of input signal is shown in Fig.15, where it can be concluded from these figures that the proposed model has better fundamental estimation even with signals has higher harmonics more than 15, and X_3 of this model still give good estimation for the total harmonics in the signal.

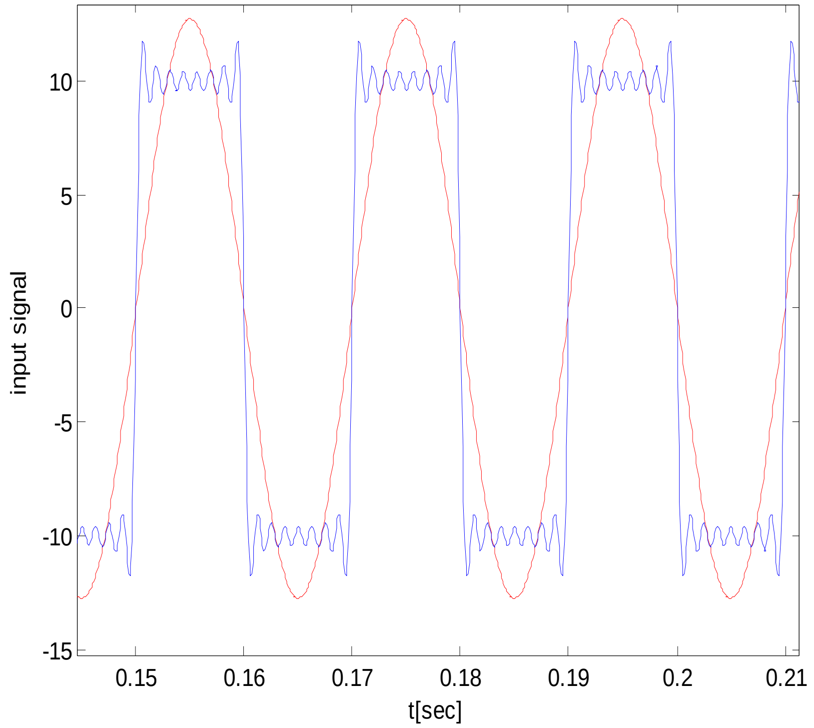


Fig.10. Input signal (blue line) and the fundamental signal in the in the input signal (red line).

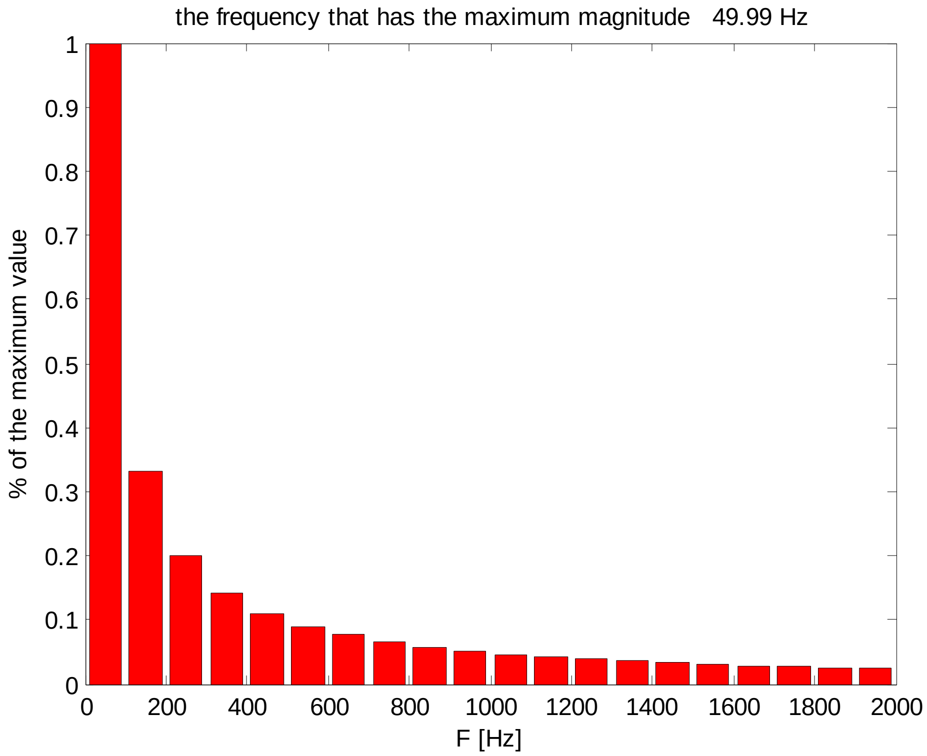


Fig.11. FFT of the input signal

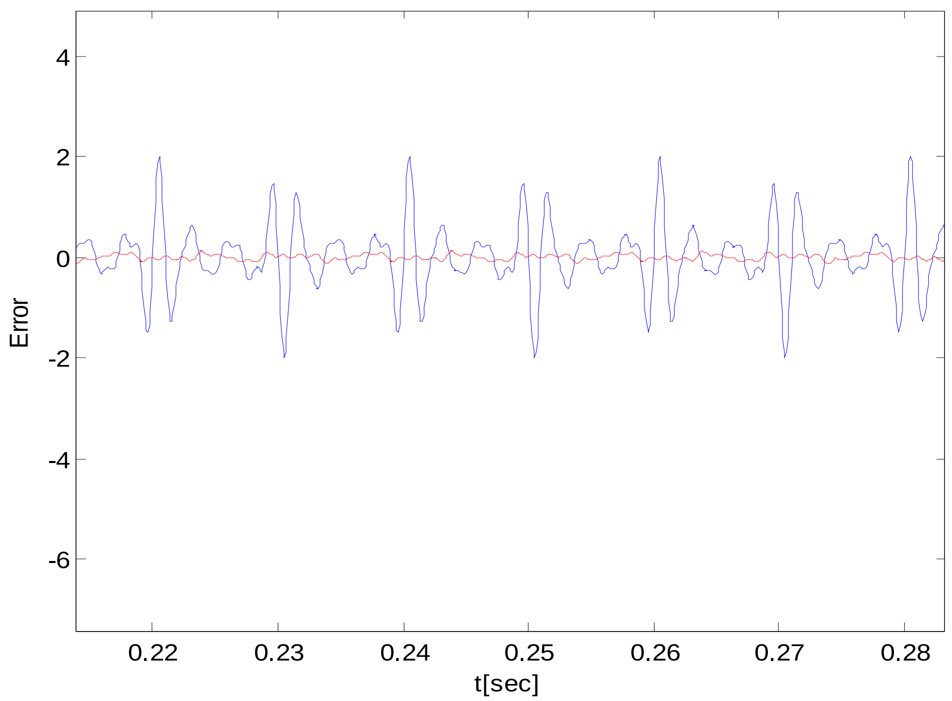
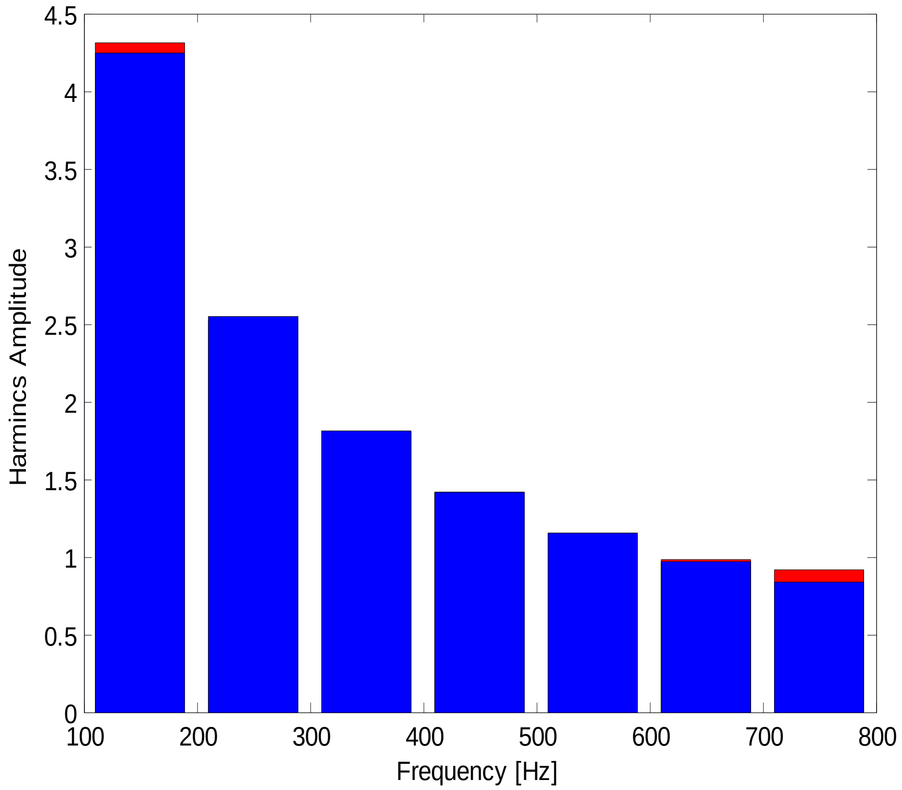
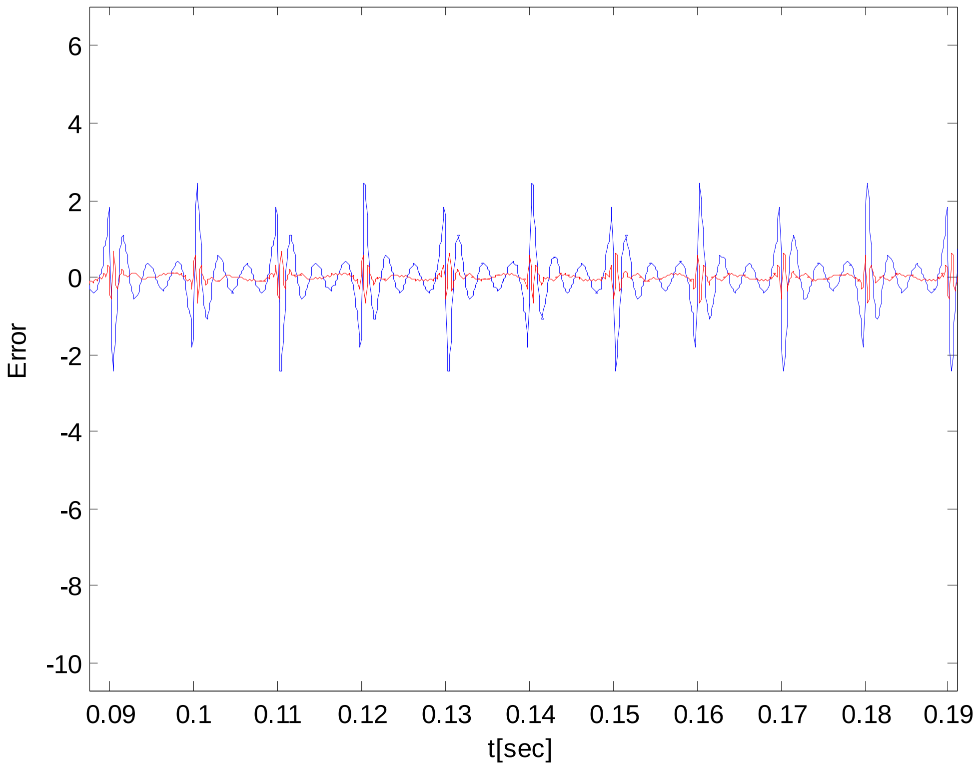


Fig.12. Error signal between the fundamental signal and the proposed model (red line) and the error between input signal and Model12 (blue line).





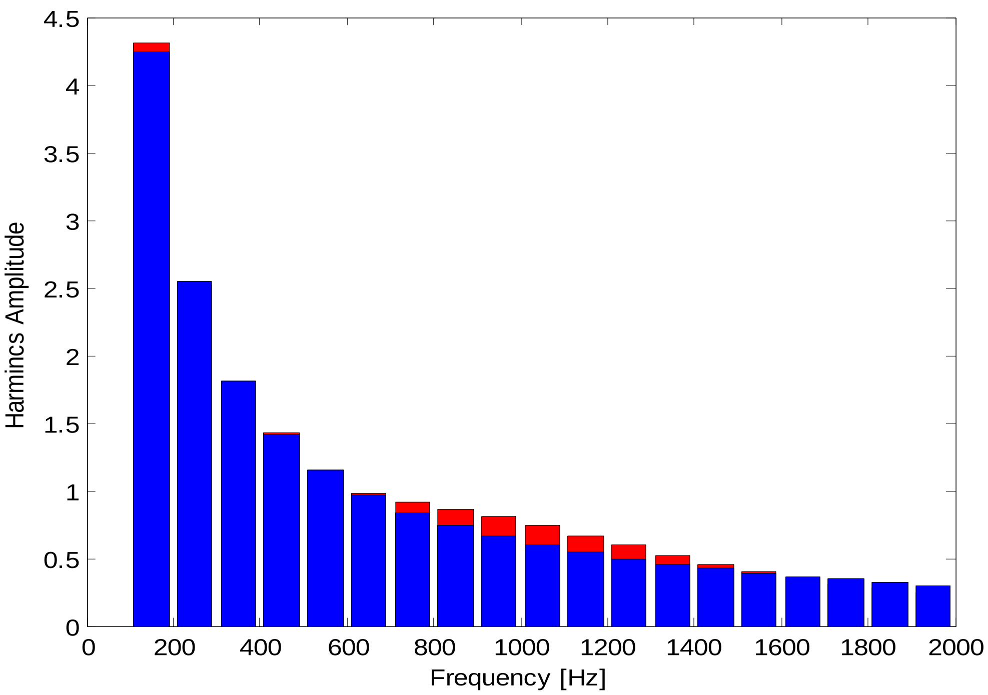


Fig.13. FFT of the harmonic component in the input signal (blue line), and for the third state variable of the proposed model (red line).

V. Conclusions

Including the harmonics in the kalman filter model improve the filter capability of estimating the fundamental component, if these harmonics are presented in the signal, and they don't have any effect if they are not existed in the input signal. A novel model which used 6 state variables to model the total harmonics in the signal for the first 15 harmonic components, instead of using 30 state variables, the proposed model showed better performance than conventional models that have more state variables.

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